



ANALYTICAL EXPRESSION OF A STEADY MHD OSCILLATORY SLIP FLOW

V. Ananthaswamy¹, M. Subha² & L. Nivedha³

¹Department of Mathematics, The Madura College, Madurai, Tamil Nadu, India.

²Department of Mathematics, MSNPM Women's College, Tamil Nadu, India.

³M. Phil., Mathematics, The Madura College, Madurai, Tamil Nadu, India.

*Corresponding Author E-mail: ananthu9777@rediffmail.com

Abstract:

In this paper we investigate the steady MHD mixed convective oscillatory flow of an electrically conducting optically thin fluid through a planner channel filled with saturated porous medium. The approximate analytical expressions for dimensionless velocities, dimensionless temperature and dimensionless concentration profiles of MHD fluid flow problem are derived. The Skin friction, the Nusselt number and Sherwood number are solved analytically and graphically.

Keywords. Chemical reaction; MHD; Oscillatory flow; Planner channel; Nusselt number

1. Introduction

The study of oscillatory fluid flow in a porous channel has been receiving considerable attention in the last recent years due to its applications in soil mechanics, ground water hydrology, irrigation, drainage, water purification processes, absorption and filtration processes in chemical engineering. Also, the Navier slip flow regime has been receiving attention of many researchers, because of its applications in modern science, technology and industrialization [1]. Owing to these applications, Makinde and Osalusi [2] presented the effects of slip conditions on the hydromagnetic steady flow in a channel with permeable boundaries.

The study of combined heat and mass transfer with chemical reaction on oscillatory fluid flow is of great practical applications in design of chemical processing equipments, formation and dispersion of flog, food processing and cooling of tower. The effects of chemical reaction on MHD oscillatory flow past through a porous plate with viscous dissipation and heat sink was presented by Vijaya et. al [3]. Rabi et. al [4] investigated the effects of chemical reaction on MHD oscillatory flow through a porous medium bounded by vertical porous plate with heat source and sore effect.

MHD has attracted the attention of many researchers and industrialists due to its rich application in cosmic fluid dynamics, meterology, motion of Earth's core and solar physics. El-Hakiem [5] has examined the influence of MHD oscillatory flow on free convection radiation through a porous medium with constant suction velocity. Makinde and Mhone [6] have investigated the problem of heat transfer to MHD oscillatory flow in a channel filled with porous medium. The effect of slip condition on unsteady MHD oscillatory flow of visous fluid in a planer channel has been analysed by Mehmood et. al [7]. The wide range of technological and industrial applications have stimulated considerable amount of interest in the study of heat an mass transfer in convection flows. Convection in porous media has application in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering device. Gholizadeh [8] has investigated the MHD oscillatory flow past a vertical porous plate through the porous medium in the presence of thermal and mass diffusion with constant heat source.

The role of thermal radiation is of major importance in engineering areas occurring at high temperatures and knowledge of radiative heat transfer becomes very important in nuclear power plants, gas turbines and the various propulsion device for aircrafts, missiles and space vehicles. Hakeem et. al [9] have examined the radiation effect of an oscillatory flow through porous medium. Srinivas et. al [10] have studied the effects of thermal radiation and space porosity on MHD mixed convection flow in a vertical channel. Chamkha [11] has analysed the significance of chemical reaction on MHD flow of a uniformly stretched vertical permeable surface. Bakr [12] have studied the effects of chemical reaction on oscillatory plate velocity and constant heat source in a rotating frame of reference.

2. Mathematical formulation of the problem

Assume the steady mixed convection, two-dimensional slip flow of an electrically conducting, heat generating, optically thin and chemically reacting oscillatory fluid flow in a planner channel filled with porous medium in the presence of thermal radiation with temperature and concentration variation. Take a Cartesian coordinate system (X, Y) where X - axis is taken along the flow and Y -axis is taken normal to the flow direction. A uniform transverse magnetic field of

magnitude B_0 is applied in the presence of thermal and solutal buoyancy effects in the direction of Y -axis. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motions are given as:

$$\frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$\frac{\partial U}{\partial t^*} = -\frac{1}{\rho} \frac{\partial P^*}{\partial X} + \nu \frac{\partial^2 U}{\partial Y^2} - \frac{\sigma_e B_0^2}{\rho} U - \frac{\nu}{K^*} U + g\beta_T(T - T_1) + g\beta_c(C - C_1) \tag{2}$$

$$\frac{\partial T}{\partial t^*} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial Y^2} - \frac{1}{\rho c_p} \frac{\partial q}{\partial Y} + \frac{Q(T - T_0)}{\rho c_p} \tag{3}$$

$$\frac{\partial C}{\partial t^*} = D \frac{\partial^2 C}{\partial y^2} - K_R(C - C_1) \tag{4}$$

The appropriate boundary conditions of the problem are

$$U = L_1 \frac{\partial U}{\partial Y}, T = T_1 + \delta_T^* \frac{\partial T}{\partial Y}, C = C_1 + \delta_c^* \frac{\partial C}{\partial Y} \text{ at } Y = 0 \tag{5}$$

$$U = 0, T = T_2 + \delta_T^* \frac{\partial T}{\partial Y}, C = C_2 + \delta_c^* \frac{\partial C}{\partial Y} \text{ at } Y = d \tag{6}$$

The radiative heat flux (Cogley et al.[17]) is given by

$$\frac{\partial q}{\partial Y} = 4(T_1 - T)I' \tag{7}$$

Introducing the following non dimensional quantities

$$x = \frac{X}{d}, y = \frac{Y}{d}, P = \frac{dP^*}{\mu U_0}, u = \frac{U}{U_0}, \theta = \frac{T - T_1}{T_2 - T_1},$$

$$\phi = \frac{C - C_1}{C_2 - C_1}, t = \frac{U_0 t^*}{d}, Re = \frac{U_0 d}{\nu}, \frac{1}{K} = \frac{d^2}{K^*},$$

$$M^2 = \frac{\sigma_e B_0^2 d^2}{\mu}, Gr = \frac{g\beta_T(T_2 - T_1)d^2}{\nu U_0}, F = \frac{4I'd^2}{k} \tag{8}$$

$$Gc = \frac{g\beta_c(C_2 - C_1)d^2}{\nu U_0}, Pe = \frac{\rho c_p U_0 d}{k}, \alpha = \frac{Qd^2}{k},$$

$$Sc = \frac{D}{U_0 d}, Kr = \frac{K_R d}{U_0}, \gamma = \frac{\gamma^*}{d}, \delta_T = \frac{\delta_T^*}{d}, \delta_c = \frac{\delta_c^*}{d}$$

In view of the above dimensionless variables, the basic field equations (2) to (4) can be expressed in non-dimensional form as

$$Re \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \left(\frac{1}{K} + M^2 \right) u + Gr\theta + Gc\phi \tag{9}$$

$$Pe \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - (F + \alpha)\theta \tag{10}$$

$$\frac{\partial \phi}{\partial t} = Sc \frac{\partial^2 \phi}{\partial y^2} - Kr\phi \tag{11}$$

The steady flow eqns. of (9), (10) and (11) are as follows:

$$A + \frac{d^2 u}{dy^2} - \left(\frac{1}{K} + M^2 \right) u + Gr\theta + Gc\phi = 0 \tag{12}$$

$$\frac{d^2\theta}{dy^2} - (F + \alpha)\theta = 0 \tag{13}$$

$$Sc \frac{d^2\phi}{dy^2} - Kr\phi = 0 \tag{14}$$

The corresponding boundary conditions are as follows:

$$u = \gamma \frac{\partial u}{\partial y}, \theta = \delta_T \frac{\partial \theta}{\partial y}, \phi = \delta_c \frac{\partial \phi}{\partial y} \text{ at } y = 0 \tag{15}$$

$$u = 0, \theta = 1 + \delta_T \frac{\partial \theta}{\partial y}, \phi = 1 + \delta_c \frac{\partial \phi}{\partial y} \text{ at } y = 0 \tag{16}$$

Where Gr means thermal Grashof number, Gc indicates solutal Grashof number, K refers permeability of the porous medium, Kr refers chemical reaction parameter, M means Hartmann number, F represents thermal radiation parameter, α represents heat source parameter, Sc indicates Schmidt number, Pe represents Peclet number, Re refers Reynolds number.

3. Solution of the non-linear boundary value problem

In recent days, a basic tool for solving non-linear problem in Homotopy analysis method (HAM) which was generated by Liao [13], is employed to solve the non-linear differential equation. The Homotopy analysis method is based on a basic concept in topology, i.e. Homotopy by Hilton [14] which is widely applied in numerical techniques as in [15-17]. Homotopy analysis method is independent of the small/large parameters not like the perturbation techniques [18]. There is a simple way to adjust and control the convergence region and rate of approximation series in Homotopy analysis method. The Homotopy analysis method has applied in many non-linear problems such as heat transfer, viscous, non-linear oscillations [19], non-linear water waves [23], etc. Such varied successful applications of the Homotopy analysis method to conform its validity for non-linear problems in science and engineering. The auxiliary parameter h is used to adjust and control the convergence of the series solution. In [27], mathematical expression is solved using the Homotopy analysis method.

In this paper, the non-linear boundary value problem which is expressed in the eqns. (12) – (16) can be solved directly. The approximate analytical expressions for the dimensionless axial velocity $u(y)$, dimensionless temperature $\theta(y)$ and dimensionless concentration profile $\phi(y)$ are as follows:

$$u(y) = C_5 \exp(\sqrt{B}y) + C_6 \exp(-\sqrt{B}y) + \left[\frac{A}{B} \right] - \left[\frac{Gr}{Z^2 - B} \right] [(C_3 \exp(Zy) + C_4 \exp(-Zy))] - \left[\frac{Gc}{N^2 - B} \right] [(C_1 \exp(Ny) + C_2 \exp(-Ny))] \tag{17}$$

$$\theta(y) = C_1 \exp(Zy) + C_2 \exp(-Zy) \tag{18}$$

$$\phi(y) = C_1 \exp(Ny) + C_2 \exp(-Ny) \tag{19}$$

Where

$$C_1 = -C_2 \left(\frac{1 + N\delta_c}{1 - N\delta_c} \right) \tag{20}$$

$$C_2 = \frac{-1}{(1 + N\delta_c)(\exp(N) - \exp(-N))} \tag{21}$$

$$C_3 = -C_4 \left(\frac{1 + Z\delta_T}{1 - Z\delta_T} \right) \tag{22}$$

$$C_4 = \frac{-1}{(1 + Z\delta_T)(\exp(Z) - \exp(-Z))} \tag{23}$$

$$C_5 = \frac{\left(-C_6(1 + \gamma\sqrt{B}) - \frac{A}{B} - \frac{Gr}{Z^2 - B}(\gamma Z(C_3 - C_4) - (C_3 + C_4)) - \frac{Gc}{N^2 - B}(\gamma N(C_1 - C_2) - (C_1 + C_2)) \right)}{(1 - \gamma\sqrt{B})} \quad (24)$$

$$C_6 = \frac{\left(-\frac{A}{B}(\exp(\sqrt{B}) - (1 - \gamma\sqrt{B})) - \frac{Gr}{Z^2 - B} \left(\exp(\sqrt{B})(\gamma Z(C_3 - C_4) - (C_3 + C_4)) + ((1 - \gamma\sqrt{B})(C_3 \exp(Z) + C_4 \exp(-Z))) \right) - \left[\frac{Gc}{N^2 - B} \right] \left(\exp(\sqrt{B})(\gamma N(C_1 - C_2) - (C_1 + C_2)) + ((1 - \gamma\sqrt{B})(C_1 \exp(N) + C_2 \exp(-N))) \right) \right)}{(\exp(\sqrt{B})(1 + \gamma\sqrt{B}) - \exp(-\sqrt{B})(1 - \gamma\sqrt{B}))} \quad (25)$$

$$B = \left(\frac{1}{K} + M^2 \right), Z = \sqrt{F + \alpha}, N = \sqrt{\frac{Kr}{Sc}} \quad (26)$$

The Skin friction (τ), the Nusselt number (Nu) and the Sherwood number (Sh) at walls $y=0$ and $y=1$ are given by

$$\tau_0 = -\left(\frac{du}{dy} \right)_{y=0} = C_5 \sqrt{B} - C_6 \sqrt{B} - \frac{Gr}{Z^2 - B}(C_3 Z - C_4 Z) - \frac{Gc}{N^2 - B}(C_1 N - C_2 N) \quad (27)$$

$$\begin{aligned} \tau_1 = -\left(\frac{du}{dy} \right)_{y=1} &= -\sqrt{B}C_5 \exp(\sqrt{B}) + \sqrt{B}C_6 \exp(-\sqrt{B}) \\ &+ \left[\frac{Gr}{Z^2 - B} \right] (ZC_3 \exp(Z) - ZC_4 \exp(-Z)) \\ &+ \left[\frac{Gc}{N^2 - B} \right] (NC_1 \exp(N) - NC_2 \exp(-N)) \end{aligned} \quad (28)$$

$$Nu_0 = -\left(\frac{d\theta}{dy} \right)_{y=0} = -(C_3 + C_4) \quad (29)$$

$$Nu_1 = -\left(\frac{d\theta}{dy} \right)_{y=1} = -(C_3 \exp(Z) + C_4 \exp(-Z)) \quad (30)$$

$$Sh_0 = -\left(\frac{d\phi}{dy} \right)_{y=0} = -(C_1 + C_2) \quad (31)$$

$$Sh_1 = -\left(\frac{d\phi}{dy} \right)_{y=1} = -(C_1 \exp(N) + C_2 \exp(-N)) \quad (32)$$

Where C_1, C_2, C_3, C_4, C_5 , and C_6 are the constants which are defined by the eqns. (20) – (25) respectively.

4. Result and discussion

Figure 1 represents the dimensionless concentration profile $\phi(y)$ versus dimensionless transverse distance y . From Fig.1(a) it is depicted that when the chemical reaction parameter Kr increases the corresponding temperature profile decreases in some fixed values of other parameters. From Fig.1(b) it is clear that when Schmidt number Sc increases the corresponding temperature profile also increases in some fixed values of other parameters. From Fig.1(c) it is clear that when Schmidt number Sc increases the corresponding temperature profile also increases in some fixed values of other parameters.

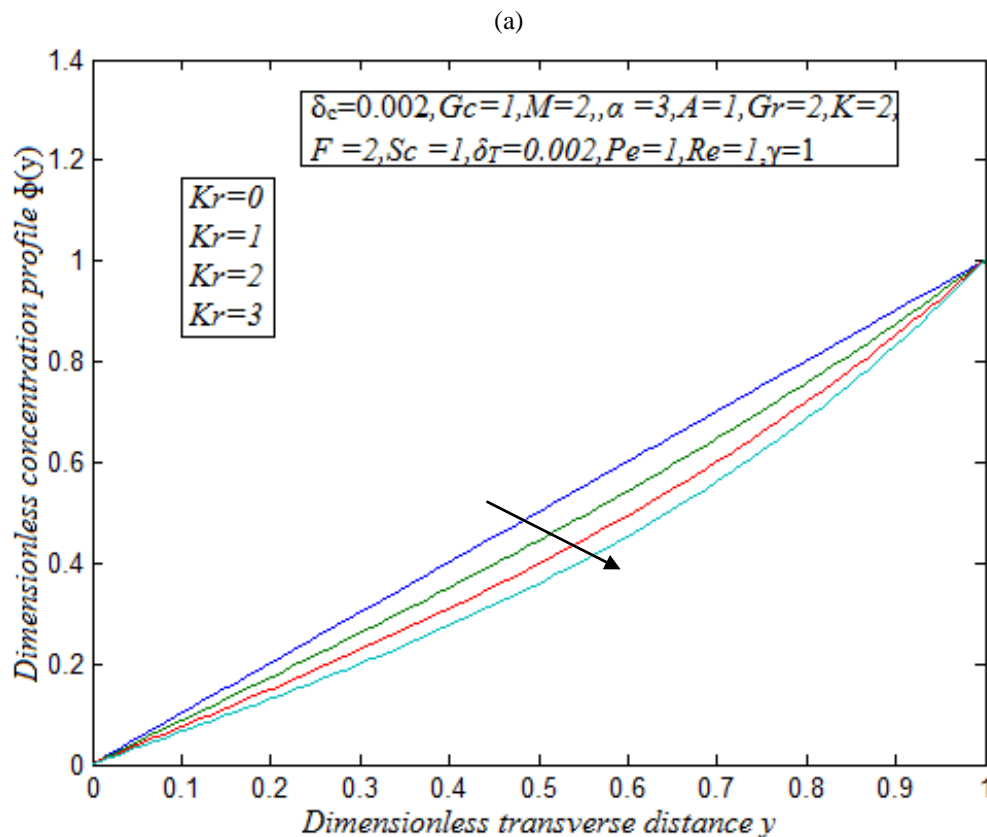
Figure 2 represents the dimensionless temperature profile $\theta(y)$ versus dimensionless transverse distance y . From Fig.2(a) it is noted that when the thermal radiation parameter F increases the corresponding temperature profile also increases in some fixed values of other parameters. From Fig.2(b) it is clear that when the heat source parameter α increases the corresponding temperature profile also increases in some fixed values of other parameters. From Fig.2(c) it is inferred that when the temperature variation parameter δ_T increases the corresponding temperature profile decreases in some fixed values of other parameters. From Fig.2(d) it is noted that when the Peclet number Pe increases the corresponding temperature profile also increases in some fixed values of other parameters.

Figure 3 represents the dimensionless velocity profile $u(y)$ versus dimensionless transverse distance y . From Fig.3(a) it is depict that when the Hartmann number H increases the corresponding dimensionless velocity profile decreases in some fixed values of other parameters. From Fig.3(b) it is noted that when the thermal Grashoff number Gr increases the corresponding velocity profile also increases in some fixed values of other parameters. From Fig 3(c) it is clear that that when the solutal Grashof number Gc increases the corresponding velocity profile also increases in some fixed values of other parameters. . From Fig.3(d) it is noted that when permeability of the porous medium K increases the corresponding velocity profile also increases in some fixed values of other parameters. . From Fig.3(e) it is clear that when the slip parameter γ increases the corresponding velocity profile also increases in some fixed values of other parameters.

Figure 4 represents the Sherwood number profile Sh versus the chemical reaction parameter Kr . From Fig.6(a) it is clear that when the Schmidt number Sc increases the corresponding Sherwood number profile Sh_0 decreases and Sherwood number profile Sh_1 increases in some fixed values of other parameters.

Figure 5 represents the Nusslet number profile Nu versus the heat source parameter α . From Fig.5(a) it is noted that when the thermal radiation parameter F increases the corresponding Nusslet number profiles Nu_0 and Nu_1 decreases in some fixed values of other parameters.

Figure 6 represents the Skin friction τ versus the thermal Grashof number Gr . From Fig.4(a) it is clear that when the slip parameter γ increases the corresponding Skin friction profile τ_0 increases and the Skin friction profile τ_1 decreases in some fixed values of other parameters.



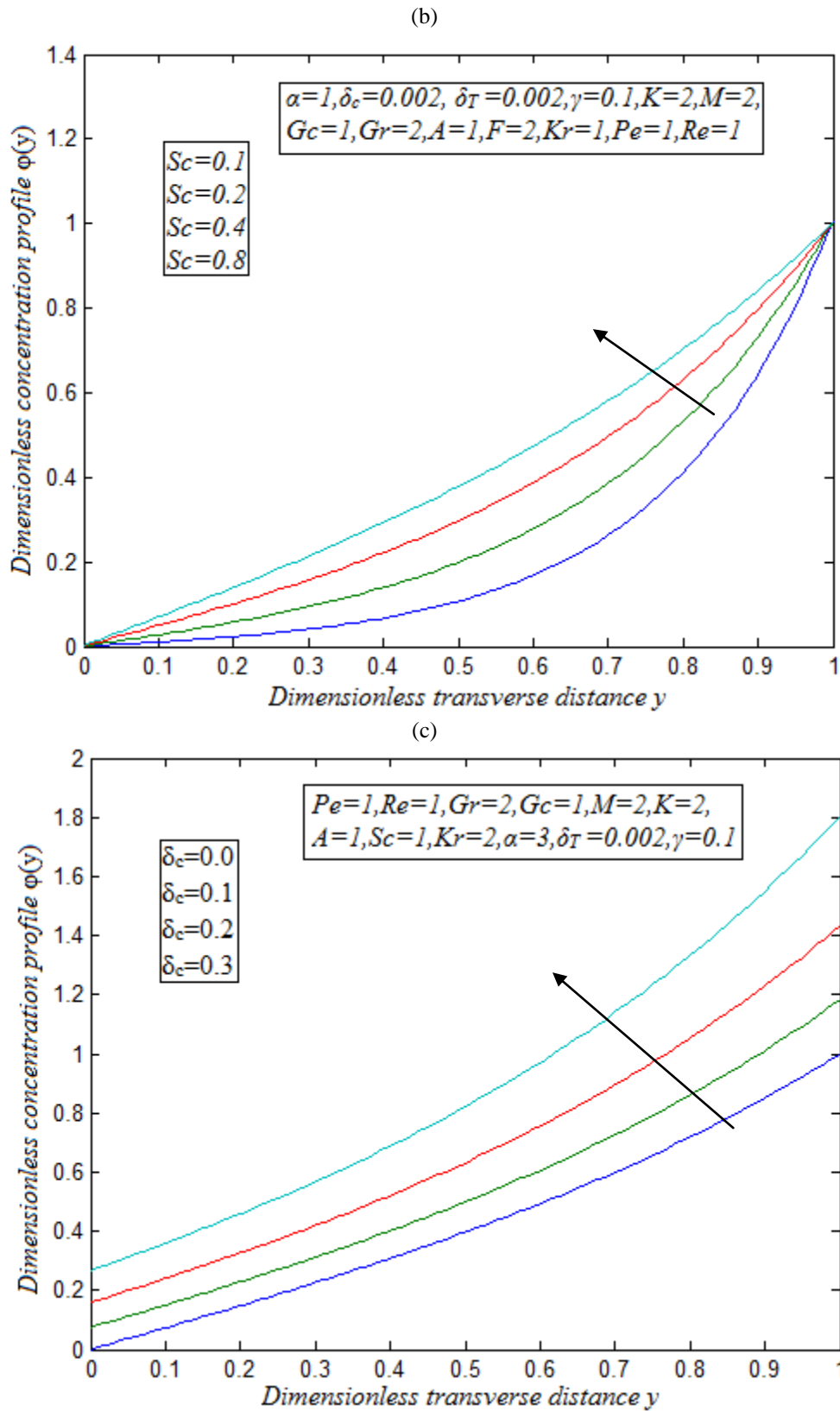
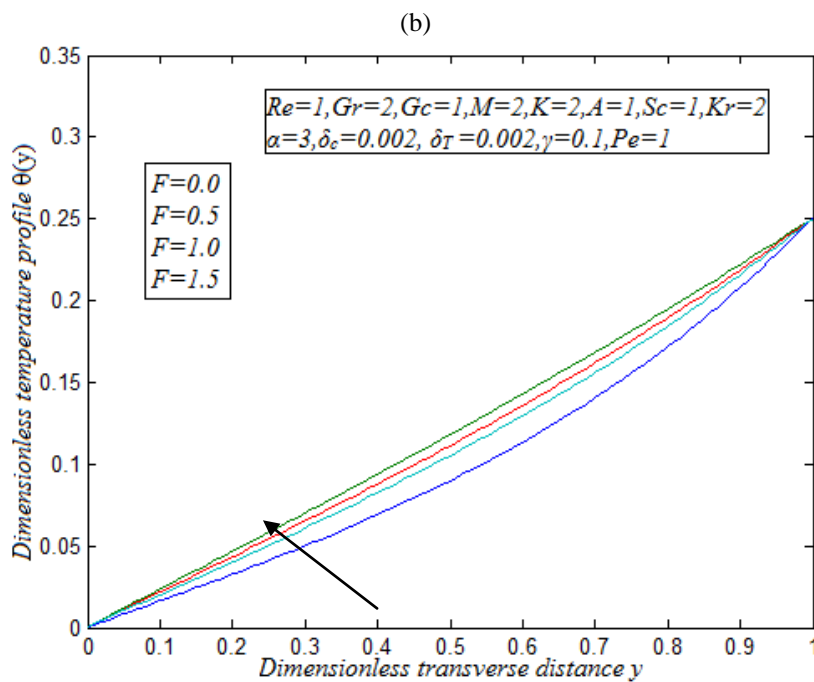
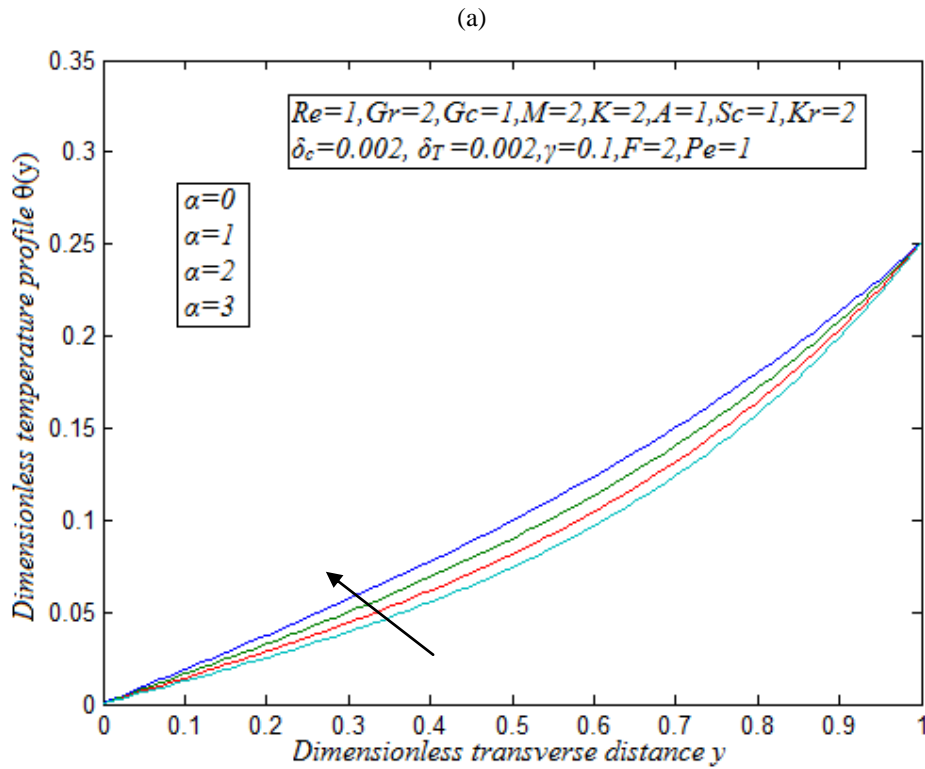


Fig.1: Dimensionless concentration profile $\phi(y)$ versus dimensionless transverse distance y . The curves are plotted using the eqn.(19) for various values of the dimensionless parameters Kr, Sc, δ_c and in some fixed values of the other dimensionless parameters, when

- (a) $\delta_c = 0.002, Gc = 1, M = 2, A = 1, Gr = 2, K = 2, \delta_T = 0.002, Pe = 1, Re = 1, F = 2, Sc = 1, \gamma = 0.1, \alpha = 3.$
- (b) $\delta_c = 0.002, Gc = 1, M = 2, A = 1, Gr = 2, K = 2, \delta_T = 0.002, Pe = 1, Re = 1, F = 2, Kr = 2, \gamma = 0.1, \alpha = 3.$
- (c) $Kr = 2, Gc = 1, M = 2, A = 1, Gr = 2, K = 2, \delta_T = 0.002, Pe = 1, Re = 1, F = 2, Sc = 1, \gamma = 0.1, \alpha = 3$



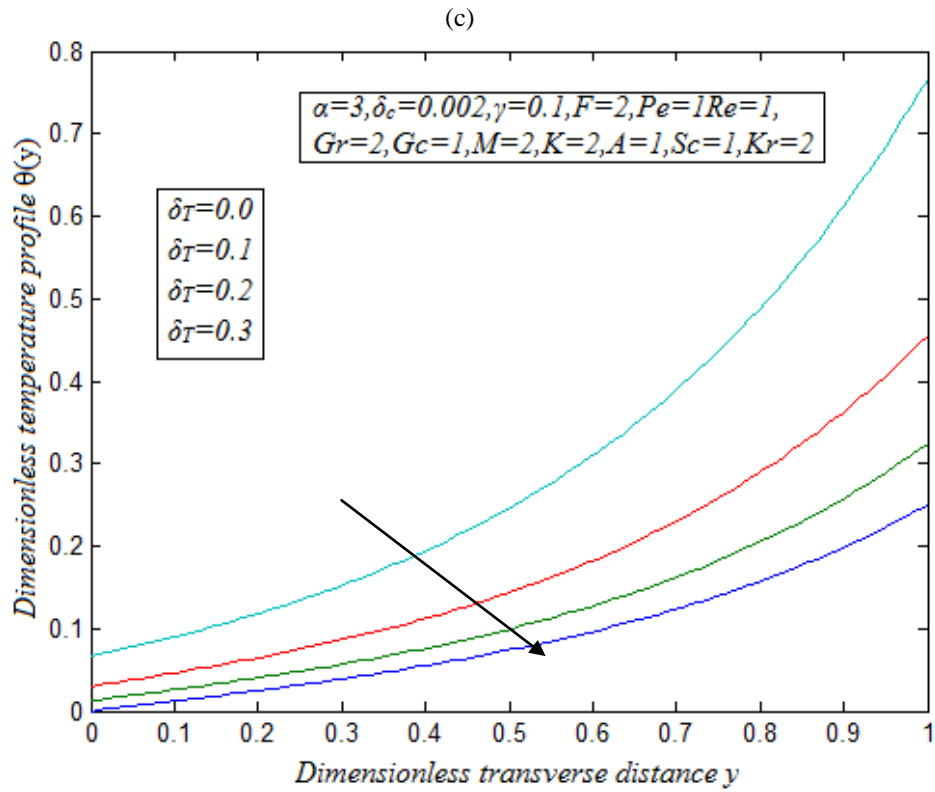
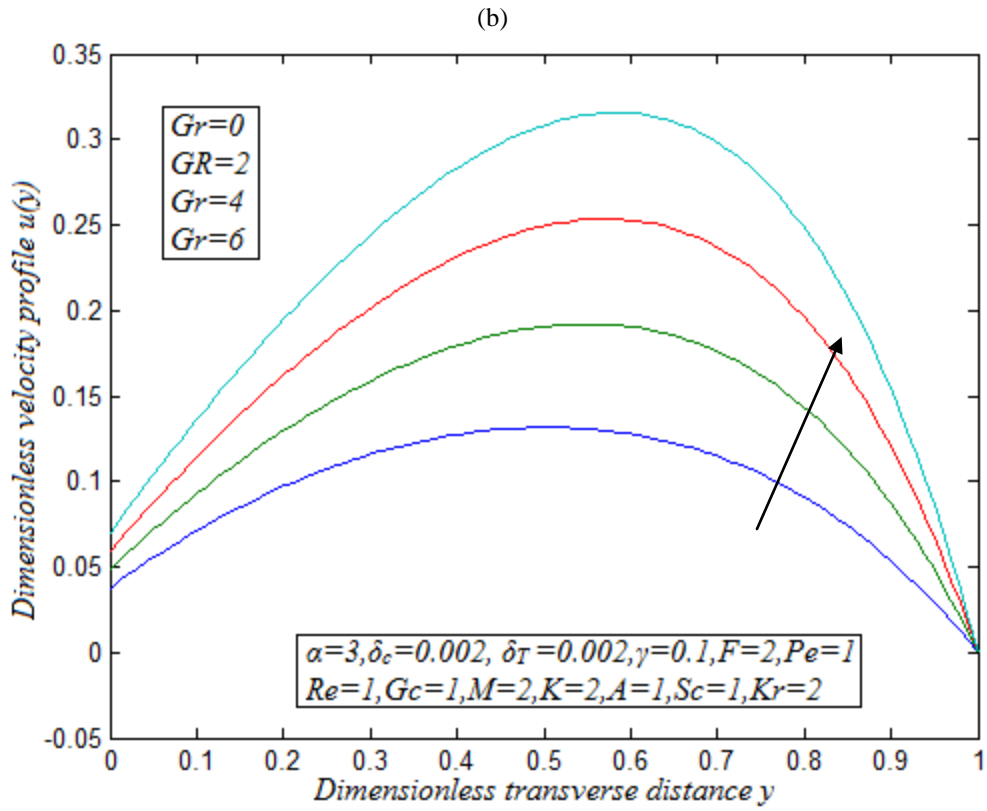
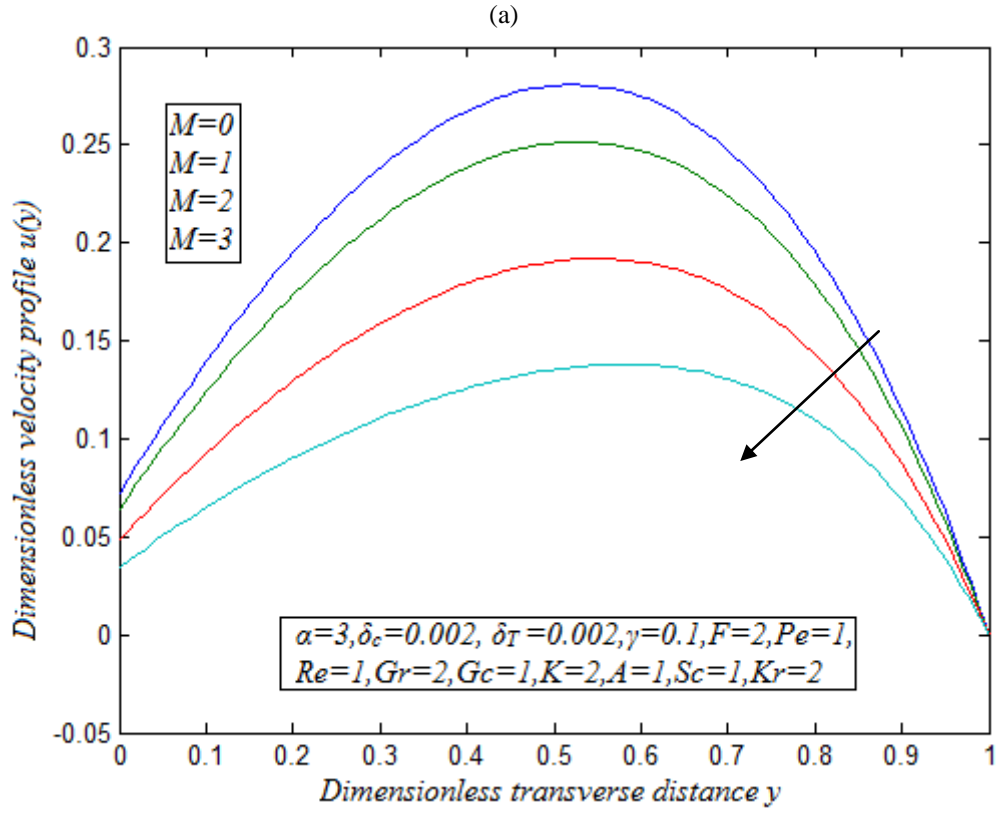
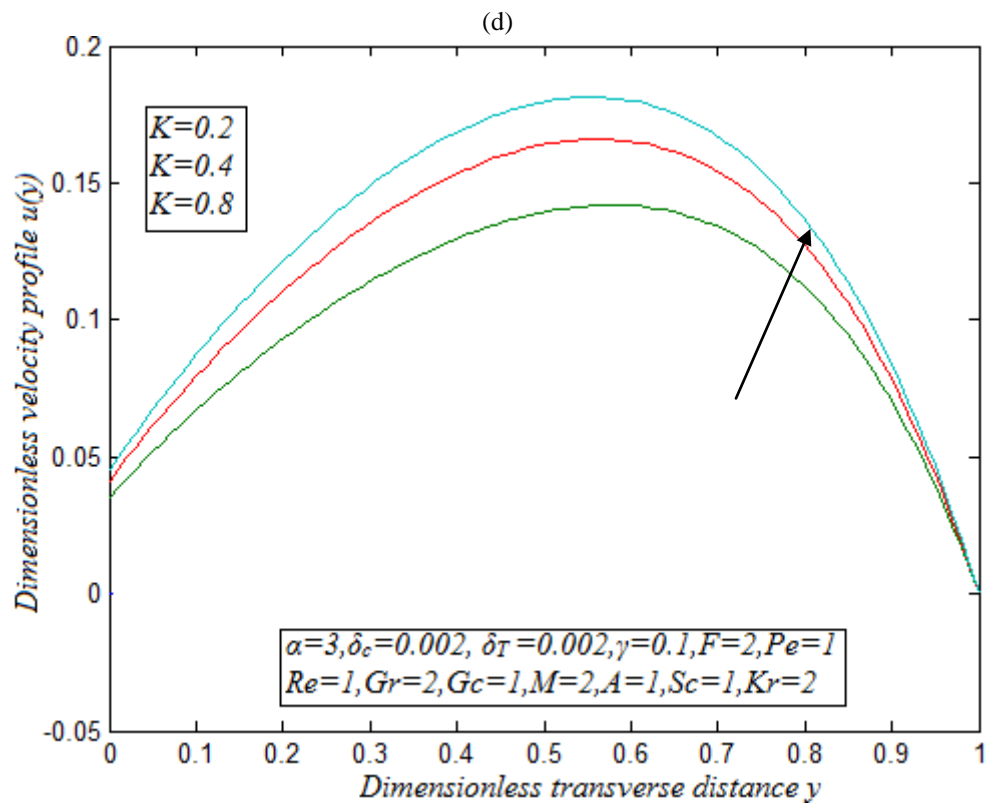
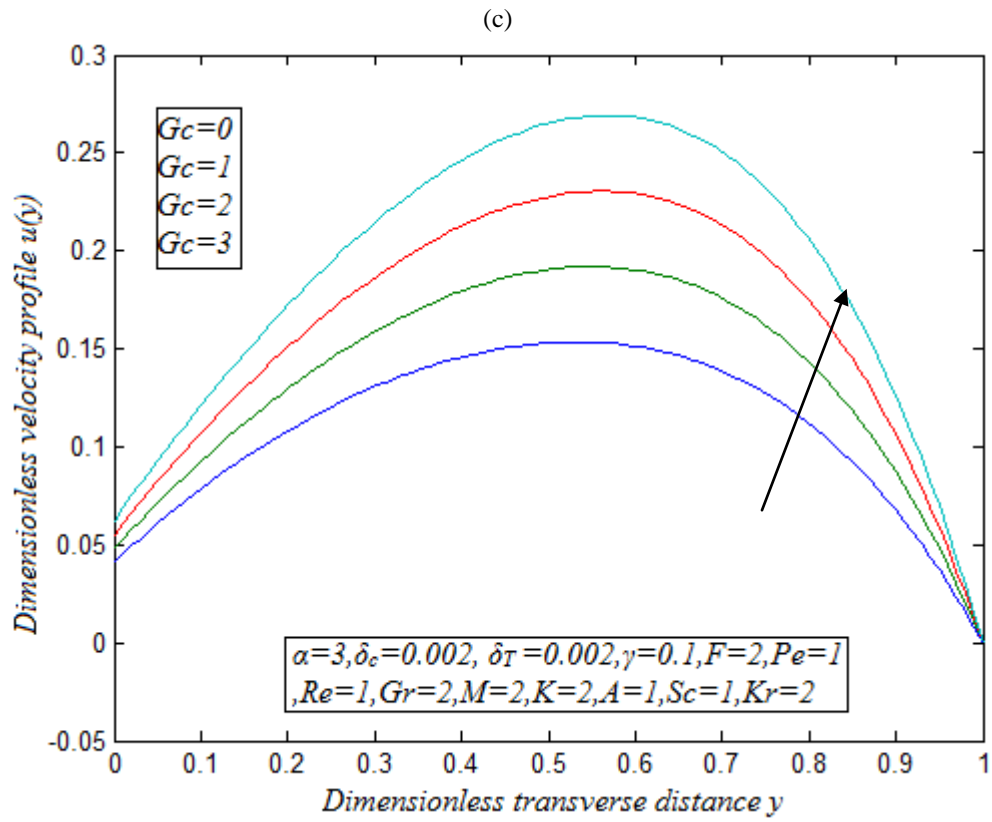


Fig.2: Dimensionless temperature profile $\theta(y)$ versus dimensionless transverse distance y . The curves are plotted using eqn.(18) for various values of the dimensionless parameter α, F, δ_T and in some fixed values of other dimensionless parameters, when

- (a) $\delta_C = 0.002, Gc = 1, M = 2, A = 1, Gr = 2, K = 2, \delta_T = 0.002, Pe = 1, Re = 1, F = 2, Sc = 1, \gamma = 0.1, Kr = 2.$
- (b) $\delta_C = 0.002, Gc = 1, M = 2, A = 1, Gr = 2, K = 2, \delta_T = 0.002, Pe = 1, Re = 1, Kr = 2, Sc = 1, \alpha = 3, \gamma = 0.1, \alpha = 3.$
- (c) $\delta_C = 0.002, Gc = 1, M = 2, A = 1, Gr = 2, K = 2, Kr = 2, Pe = 1, Re = 1, F = 2, Sc = 1, \gamma = 0.1, \alpha = 3.$





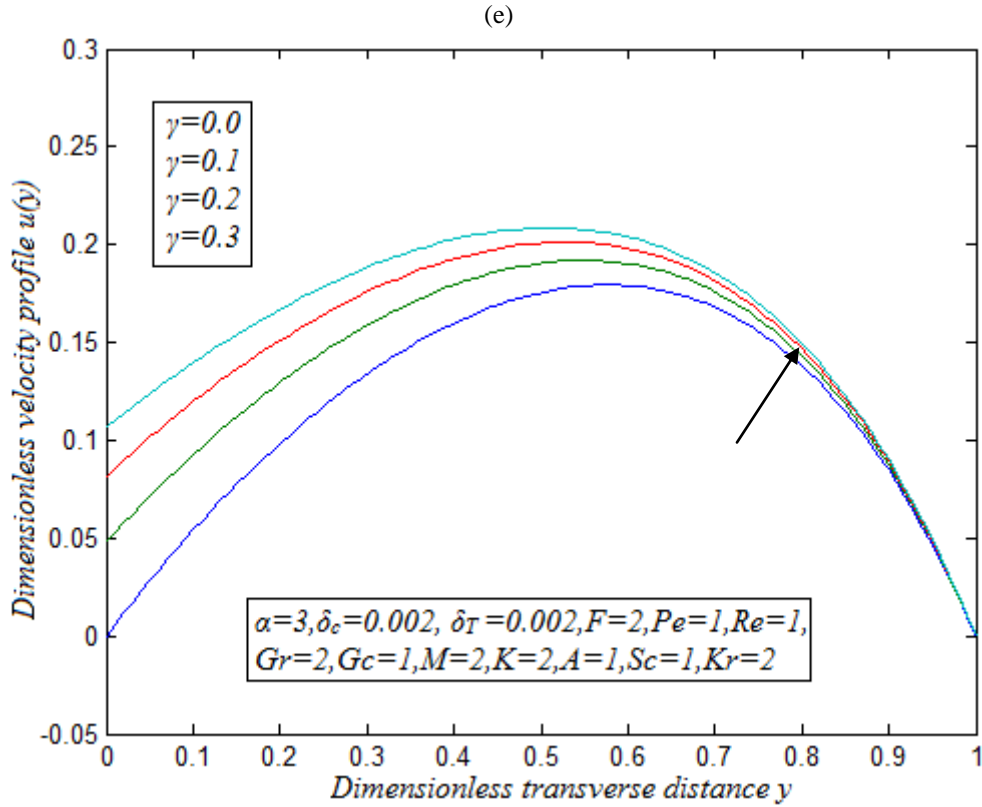


Fig.3: The dimensionless velocity profile $u(y)$ versus dimensionless transverse distance y . The curves are plotted using eqn.(17) for various values of the dimensionless parameter M, Gr, Gc, K, γ and in $\gamma = 0.1, \alpha = 3$ some fixed values of the other dimensionless parameters, when

- (a) $\delta_c = 0.002, Gc = 1, A = 1, Gr = 2, K = 2, \delta_T = 0.002, Pe = 1, Re = 1, F = 2, Sc = 1, Kr = 2$
 $\gamma = 0.1, \alpha = 3$.
- (b) $\delta_c = 0.002, Gc = 1, A = 1, K = 2, \delta_T = 0.002, Pe = 1, Re = 1, F = 2, Sc = 1, Kr = 2, M = 2$
 $\gamma = 0.1, \alpha = 3$.
- (c) $\delta_c = 0.002, A = 1, Gr = 2, K = 2, \delta_T = 0.002, Pe = 1, Re = 1, F = 2, Sc = 1, Kr = 2, M = 2$
 $\gamma = 0.1, \alpha = 3$.
- (d) $\delta_c = 0.002, Gc = 1, A = 1, Gr = 2, \delta_T = 0.002, Pe = 1, Re = 1, F = 2, Sc = 1, Kr = 2, M = 2$
 $\gamma = 0.1, \alpha = 3$.
- (e) $\delta_c = 0.002, Gc = 1, A = 1, Gr = 2, K = 2, \delta_T = 0.002, Pe = 1, Re = 1, F = 2, Sc = 1, Kr = 2$
 $M = 2, \alpha = 3$.

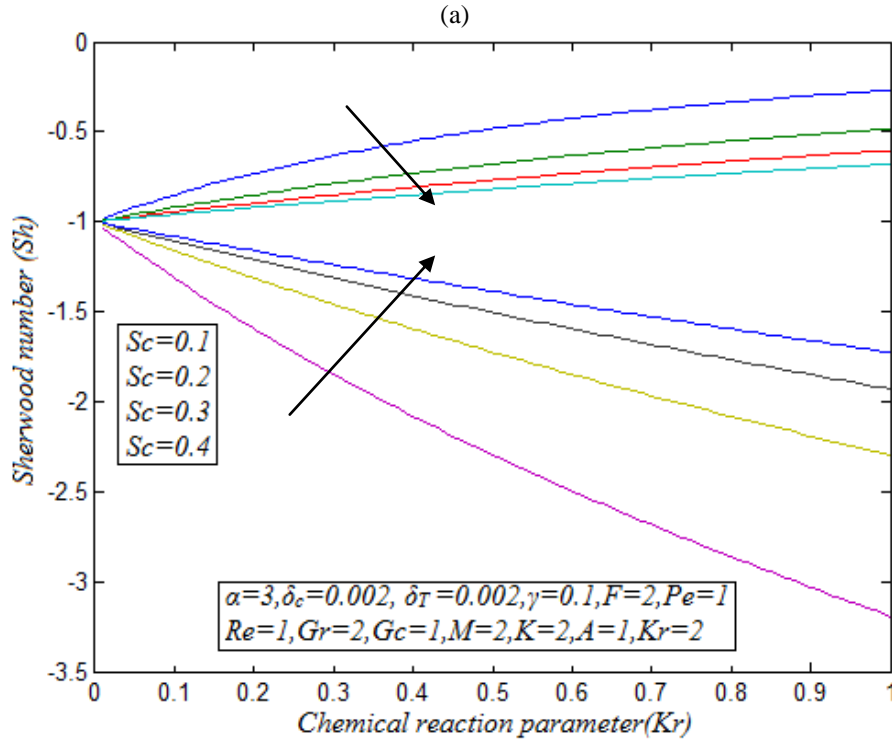


Fig.4: Sherwood number profile Sh versus the chemical reaction parameter Kr . The curves are plotted using the eqns.(31) and (32) for various values of Schmidt number Sc and in some fixed values of other dimensionless parameters, when

(a) $\delta_c = 0.002, Gc = 1, M = 2, A = 1, Gr = 2, K = 2, \delta_T = 0.002, Pe = 1, Re = 1, F = 2, Kr = 2,$
 $\gamma = 0.1, \alpha = 3.$

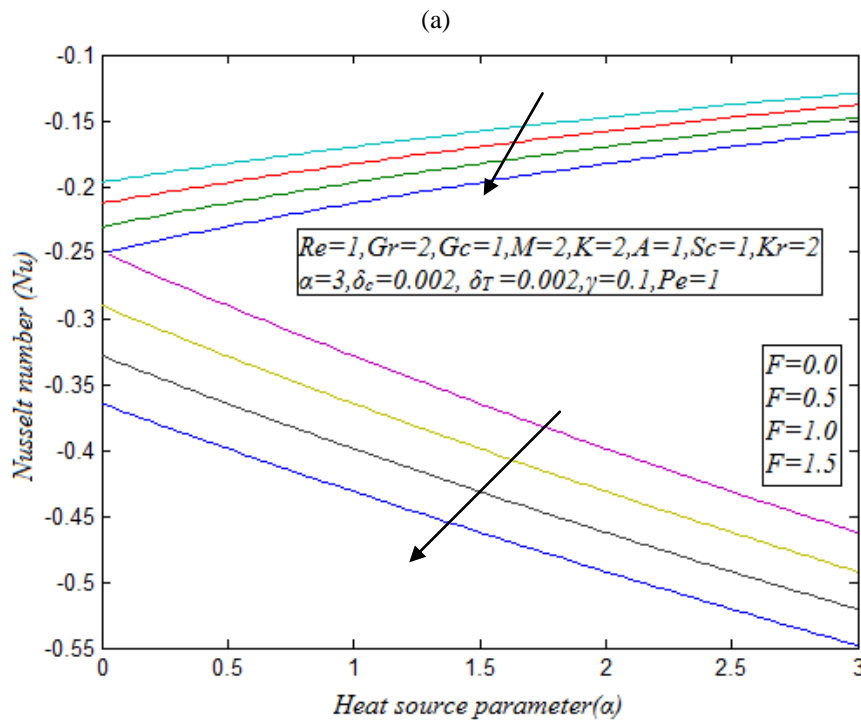


Fig.5: Nusselt number profile Nu versus the heat source parameter α . The curves are plotted using the eqns.(29) and (30) for various values of Thermal radiation parameter F and in some fixed values of other dimensionless parameters, when

(a) $\delta_c = 0.002, Gc = 1, M = 2, A = 1, Gr = 2, K = 2, \delta_T = 0.002, Pe = 1, Re = 1, Sc = 1, Kr = 2,$
 $\gamma = 0.1, \alpha = 3.$

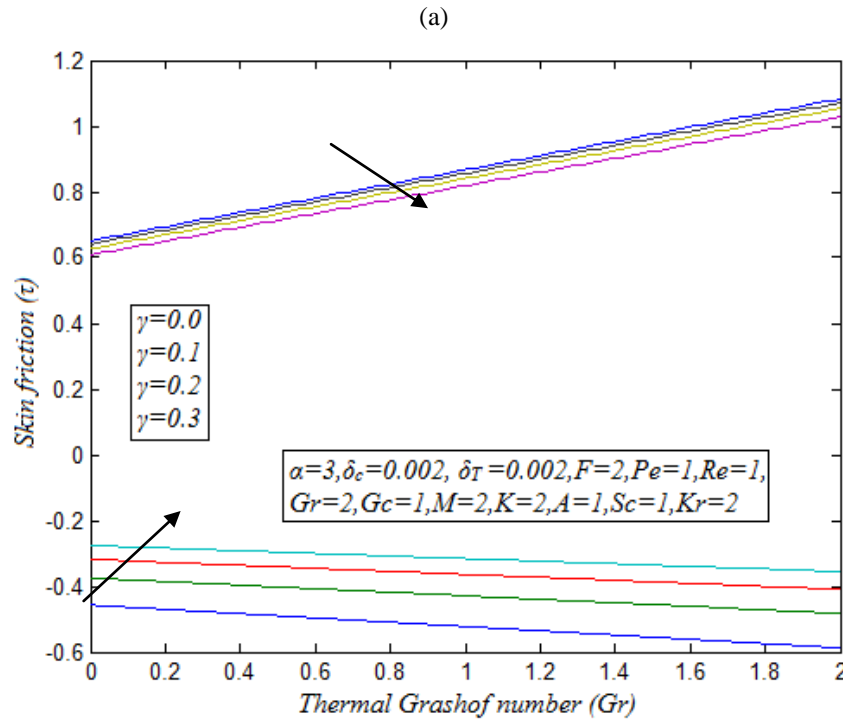


Fig.6: Skin friction τ versus the thermal Grashof number Gr . The curves are plotted using the eqns.(27) and (28) for various values of Schmidt number Sc and in some fixed values of other dimensionless parameters, when
 (a) $\delta_c = 0.002, G_c = 1, M = 2, A = 1, Gr = 2, K = 2, \delta_T = 0.002, Pe = 1, Re = 1, F = 2, Kr = 2,$
 $\gamma = 0.1, \alpha = 3$

5. Conclusion

The analytical expressions of the dimensionless velocities, dimensionless temperatures and dimensionless concentration profiles for the steady MHD fluid flow problem are derived mathematically and graphically. We conclude from the plotted velocity profile as fluid velocity increases while increasing thermal Grashof number Gr , thermal Radiation parameter F , permeability of the Porous medium K , solutal grashof number Gc and it decreases with increasing of Hartmann number M and also temperature profile increases while increasing thermal Radiation parameter F , heat source parameter α and it decreases with increasing of temperature variation parameter δ_T . and also concentration profile increases while increasing Schmidt number Sc and concentration variation parameter δ_c and it decreases with increasing chemical reaction parameter Kr .

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Appendix-A: Nomenclature

U, V	Velocity components in X, Y resp.
d	Dimensional channel width
t^*	Dimensional time
C_p	Specific heat at constant pressure
P^*	Dimensional pressure
K^*	Dimensional porous permeability coefficient
g	Gravitational force
T	Dimensional fluid temperature
T_1, T_2	Wall temperatures

k	Thermal conductivity
q	Radiative heat flux
Q	Dimensional heat source parameter
C	Dimensional fluid concentration
C_1, C_2	Wall concentrations
D	Mass diffusivity
K_R	Dimensional chemical reaction parameter
δ_C	Concentration variation parameter
m_1	Maxwell's reflection coefficient
$K_{\lambda,w}$	Radiation absorption coefficient at the wall
e_{b,λ_r}	Planck's function
Re	Reynolds number
M^2	Hartmann number
K	Permeability of the porous medium
Gr	Thermal grashof number
Gc	Solutal grashof number
Pe	Peclet number
F	Thermal radiation parameter
Sc	Schmidt number
Kr	Chemical reaction parameter
A	Real constant
μ	Dynamic viscosity
ρ	Fluid density
ν	Kinematic viscosity coefficient
σ_e	Electric conductivity of the fluid
β_T	Volumetric thermal expansion
β_C	Volumetric concentration expansion
α	Heat source parameter
ω	Frequency of the oscillation
γ	Slip parameter
δ_T	Temperature variation parameter