STAR
Research Journal
Available online at www.starresearchjournal.com (Star International Journal)
MATHEMATICS

APPROXIMATE ANALYTICAL SOLUTION FOR MHD FLOW OVER A NONLINEAR POROUS STRETCHING SHEET

V. Ananthaswamy[1] & P. Sivapreethi[2]

[1,2]Department of Mathematics, The Madura College, Madurai, Tamil Nadu, India.
[2]M. Phil., Mathematics, The Madura College, Madurai, Tamil Nadu, India.
[*]Corresponding Author E-mail: ananthu9777@rediffmail.com

Abstract*:*

In this paper, the MagnetoHydroDynamic (MHD) boundary layer flow over a nonlinear porous stretching sheet is investigated by employing the Homotopy Analysis Method (HAM). The numerical solution of the governing non-linear problem is developed.. Graphical results have been presented and discussed for the pertinent parameters. The Homotopy analysis method can be easily extended to solve the non-linear boundary value problems in MHD fluid flow problems. Our analytical and graphical results are compared with the previous works and a satisfactory agreement is noted.

Keywords. MHD boundary layer equation; Nonlinear porous stretching sheet; Homotopy analysis method.

1. Introduction

Recently, many scientist and engineers have paid more attention on new methods for solving boundary layer equations, arising from mathematical modeling of fluid mechanics problems Abdou et al. [1]. In 1973, McCormack et al.[3] introduced the stretching sheet problem. The stretching problems for steady flow have been extensively in various aspects, for example to non-Newtonian fluids, MHD flows, porous plate, porous medium and heat transfer analysis. The literature on the topic is quite extensive and hence cannot be described here in detail. In the last several years Raftari et al. [4] have considerable attention has been given to the study of the hydrodynamic thermal convection due to its numerous applications in geophysics. Chakrabarti et al.[6] developed the Hydromagnatic flows become important due to industrial applications. Similarly Sheet stretching is an important operation in polymer industry. This is not surprising realizing the fact that most studies carried out in the past on MHD flows of non-Newtonian fluids were concerned primarily with simple rheological models such as second-order model, or third-order model at best.

However, some most recent works of eminent researchers regarding the flow over a stretching sheet may be mentioned in the Mukhopadhyay [9]. Although the numerical approach allows studying more complex boundary conditions, the importance of analytical solutions is undeniable and it is witnessed by the large amount of work performed, particularly in recent years, on this subject. Various kinds of solutions methods were used to handle the boundary layer problem. Recently Liao [13]-[18] developed the Homotopy analysis method (HAM) by combining the standard Homotopy analysis and Laplace transformation method for solving nonlinear problems. This method is very well suited to physical problems since it does not require unnecessary linearization, discretization and other restrictive methods and assumptions which may change the problem being solved, sometimes seriously. From continuity of stretching sheet solution would induce a far field suction towards the sheet, while the stretching sheet would cause a velocity away from the sheet. Thus from the physical velocity of the stretching sheet is not confined within the boundary layer and the flow unlikely to exist unless adequate suction on the boundary is imposed.

Vasu et al.[24] introduced the basic motivation of the present study is to extend our previous approach proposed in to solve MHD boundary layer problem over a nonlinear porous stretching sheet on semi-infinite domain..To the best of authors knowledge no attempt has been madeto exploit this method to solve nonlinear equation on semi-infinite domain. Also our aim in this article is to compare the results with solutions to the existing ones .The laminar boundary layer flow of a viscous incompressible fluid and heat transfer towards a stretching cylinder embedded in a porous medium. Rao et al. [26] examined the effects of viscous dissipation and joule heating on magneto hydrodynamics flow of a fluid with variable properties past a stretching vertical plate. The heat transfer due to stretching porous sheet in presence of heat source concluded the effect of viscous dissipation on flow over a stretching porous sheet in presence of heat source.

2. Formulation of the problem

Let us consider the MHD flow of an incompressible viscous fluid over a non-linear porous stretching sheet at $y = 0$. The fluid is electrically conducting under the influence of an applied magnetic field $B(x)$ normal to the stretching sheet. The induced magnetic field is neglected. The resulting boundary-layer equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\sigma\frac{B_0{}^2(x)}{\rho}u \quad (2)$$

Here u and v are the velocity components in the x - and y -directions respectively, ν is the kinematic viscosity, ρ is the density and σ is the electrical conductivity of the fluid. In eqn. (2), the external electric field and polarization effects are negligible, therefore

$$B(x)=B_0 x^{\frac{n-1}{2}} \quad (3)$$

The boundary conditions corresponding to the non-linear porous stretching sheet are given below

$$u(x,0)=cx^n, v(x,0)=-V_0 \quad (4)$$
$$u(x,y)\to 0\, as\, y\to\infty$$

Where c is the stretching parameter V_0 is the porosity of the plate (where v_0 >0 corresponds to suction and v_0 <0corresponds to injection) . Upon making use of the following substitutions

$$\eta=\sqrt{\frac{c(n+1)}{2\nu}}x^{\frac{n-1}{2}}y,\ u=cx^n f'(\eta) \quad (5)$$

$$dv=-\sqrt{\frac{c\nu(n+1)}{2}}x^{\frac{n-1}{2}}\left[f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right]$$

The resulting non-linear differential equation and boundary conditions are of the following form

$$f'''+ff''-\beta f'^2-Mf'=0 \quad (6)$$
$$f(0)=K,\ f'(0)=1,\ f'(\infty)=0$$

Where

$$\beta=\frac{2n}{n+1},\ M=\frac{2\sigma B_0{}^2}{\rho c(1+n)},\ \ K=\frac{V_0}{\sqrt{\frac{c\nu(n+1)}{2}x^{\frac{n=1}{2}}}} \quad (7)$$

β is the non-dimensional parameter, M is the magnetic parameter, and K is the wall mass transfer parameter.

3. Solution of the problems using the Homotopy analysis method

This paper deals with a basic strong analytic tool for nonlinear problems, namely the Homotopy analysis method (HAM) which was generated by Liao [14]-[16], is employed to solve the nonlinear differential eqns. (6) - (7). The Homotopy analysis method is based on a basic concept in topology. Unlike perturbation techniques like, the Homotopy analysis method is independent of the small/large parameters. Unlike all other reported perturbation and non-perturbation techniques such as the artificial small parameter method, the δ-expansion method and Adomian decomposition method, the Homotopy analysis method provides us a simple way to adjust and control the convergence region and rate of approximation series. As seen in the non-linearity present in electo hydrodynamic flow takes the form of a rational function, and thus, poses a greater challenge with respect to finding approximate solutions analytically. Our results show that even in this case, HAM yields excellent results.

$$f(\eta)=K\left(2-e^{\frac{-x}{K}}\right)-h\left(C_1\frac{x^2}{2}+C_2x+C_3-K^3\left[(2+M)e^{\frac{-x}{k}}+\frac{(\beta-1)}{8}e^{\frac{-2x}{K}}\right]\right) \quad (8)$$

Where

$$C_1=0 \quad (9)$$

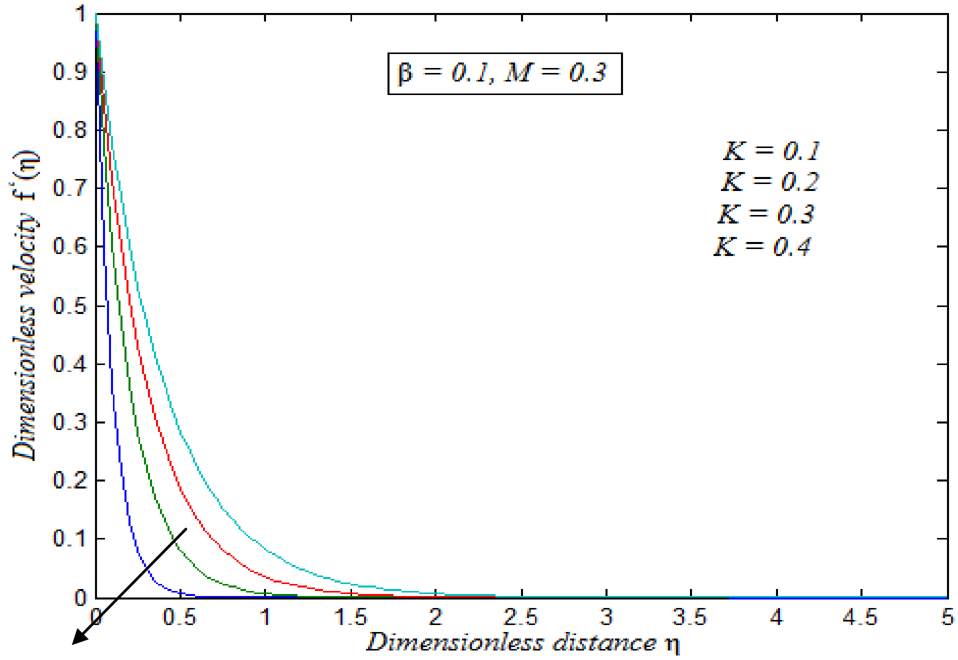
$$C_2=-K^2\left(2+M+\frac{(\beta-1)}{4}\right) \quad (10)$$

$$C_3=K^3\left[2+M+\frac{(\beta-1)}{8}\right] \quad (11)$$

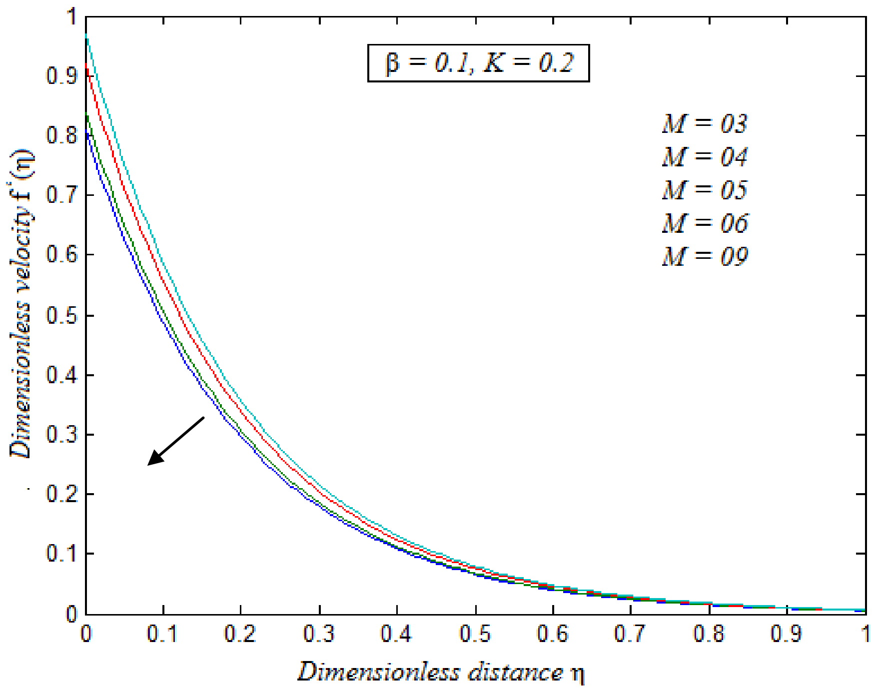
4. Results and discussion

Figure 1 represents the dimensionless distance η versus the dimensionless velocity $f'(\eta)$. From Fig. 1(a) , it is noted that when the effects of the mass suction parameter K decreases the corresponding velocity profile also increases in some fixed values of the other parameters β, M. From Fig. 1(b), it is inferred that when the magnetic parameter M decreases the corresponding dimensionless velocity profile increases in some fixed values of the other parameters β, K. From Fig. 1(c), it is depict that when the non-dimensional parameter β increases the corresponding dimensionless velocity profile also decreases in some fixed values of the other parameter M, K.

(a)



(b)



(c)

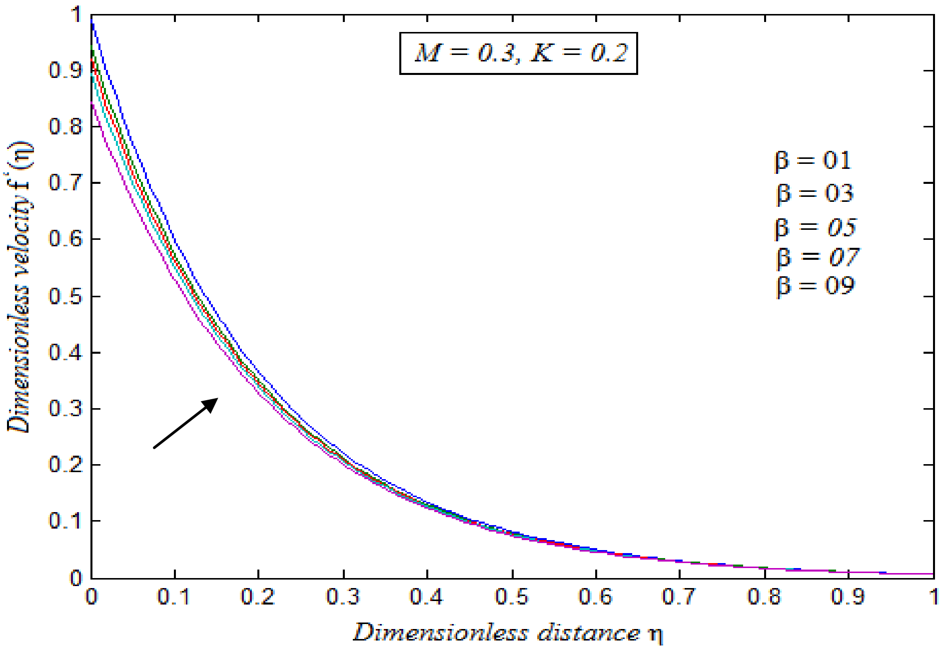


Fig. 2: Dimensionless distance η versus the dimensionless velocity $f'(\eta)$. The curves are plotted using the eqn.(8) for various values of the dimensionless parameters β, M, K and in some fixed values of the other dimensionless parameters, when

$(a)\beta = 0.1, M = 0.3;\quad (b)\beta = 0.1, K = 0.2;\quad (c) M = 0.3, K = 0.2;$

Table 1 Comparison of the numerical value of $f''(0) = \alpha$

K	β	M	Croco Transformation		Shooting Method	
			Previous work	Current work	Previous work	Current work
0	1.5	0	-1.4902	-1.4900	-1.4860	-1.4860
		1	-1.5253	-1.5253	-1.52527	-1.52529
		5	-2.51616	-2.51617	-2.5162	-2.5164
		10	-3.36632	-3.36633	-3.36631	-3.36635
		50	-7.16471	-7.16472	-7.16471	-7.16473
		100	-10.0664	-10.0663	-10.0664	-10.0665

Table 2 Comparison of the numerical value of $f''(0) = \alpha$

K	β	M	Croco Transformation		Shooting Method	
			Previous work	Current work	Previous work	Current work
0	5	0	-1.90433	-1.90433	-1.9025	-1.9026
		1	-2.15344	-2.15345	-2.1529	-2.1529
		5	-2.94150	-2.94150	-2.94144	-2.94145
		10	-3.69567	-3.69568	-3.6956	-3.6956

		50	-7.32561	-7.32561	-7.3256	-7.3258
		100	-10.1816	-10.1818	-10.1816	-10.1816

5. Conclusion

In this paper the modified Homotopy analysis method is employed to get the analytical solution for the MHD flow over a nonlinear porous stretching sheet is derived mathematically and graphically. we describe the analysis for third order nonlinear differential equation over an infinite interval arising in MHD boundary layer flow. The semi-numerical schemes described here offer advantages over solutions obtained by HAM and numerical methods etc. The results are presented in Tables and the effects of the emerging parameters are discussed numerically.

Acknowledgement

Researchers express their sincere gratitude to the secretary Shri. S. Natanagopal, The Madura College Board, Madurai, Dr. J. Suresh, The Principal and Dr. C. Thangapandi, Head of the Department of Mathematics, The Madura College, Madurai, Tamil Nadu, India for their constant support and encouragement.

References

1. M.A. Abdou, S. khan, K. Yasir, M. Faraz Naeem, Q. Wu, Numerical solution of MHD flow over a nonlinear porous stretching sheet,Vol. 31, 3, (2012):547-549.
2. B.C. Sakiadis, Boundary-layer behavior on continuous solid surfaces, AIChE J., **7**(1)(1961):26-28.
3. P.D.McCormack, L. Crane , Physics of fluid dynamics, New York, Academic Press, (1973):429-435.
4. B. Raftari, A. Yildirim, The application of Homotopy perturbation method for MHD flows of UCM fluids above porous stretching sheets, Computers &Mathematics with Applications, **59**, (2010):3328.
5. E. M. Abo-Eldahab, A.M. Salem, A. M., Hall effect on MHD free convection flow of a non-Newtonian power law fluid at a stretching surface, Int. Comm. Heat Mass Transfer, 31(3)(2004): 343-354.
6. A. Chakrabarti, A.S. Gupta, Hydromagnetic flow and heat transfer over a stretching sheet, Q Appl Math, 37(1)(1979): 73-78.
7. K. Vajravelu, A. Hadjinicolaou, Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation, Int. Comm. Heat Mass Transfer, 20(3)(1993): 417- 430,
8. M. S. Abel, S. K. Khan, K. V. Prasad, Convective heat and mass transfer in a visco-elastic fluid flow through a porous medium over a stretching sheet, Int. J. Numer Meth Heat Fluid Flow, 11 (2001), 8, pp. 779-792
9. S. Mukhopadhyay, Analysis of boundary layer flow and heat transfer along a stretching cylinder in a porous medium, ISRN Thermodynamics Volume 2012 (2012), Article ID 704984, 7 pages , http://dx.doi.org/10.5402/2012.
10. K. K. Jaber, Effect of viscous dissipation and joule heating on MHD flow of a fluid with variable properties past a stretching vertical plate, European Scientific Journal, 10(33)(2014):35-38.
11. G. Mandal, and K. K. Mandal, Effect of hall current on MHD couette flow between thick arbitrarily conducting plates in a rotating system, J. Physical Soc. Japan, 52(1983): 470-477.
12. H. Kumar, Heat transfer over a stretching porous sheet subjected to power law heat flux in presence of heat source, Thermal Science, 15(Suppl. 2)(2011): S187-S194.
13. S.J. Liao and A.T.Chwang, Application of Homotopy analysis method in nonlinear oscillations, Trans. ASME: J. Appl.Mech., 65(1998) ,914-922.
14. S.J. Liao, An analytic approximate technique for free oscillations of positively damped systems with algebraically decaying amplitude, Intl J. Non-Linear Mech., 38(2003), 1173-1183.
15. S.J. Liao, An analytic approximation of the drag coefficient for the viscous flow past a sphere, Intl J. Non-Linear Mech., 37(2002), 1-18.
16. S.J. Liao, The Homotopy analysis method in non-linear differential equations, Springer and Higher Educattion Press, (2012):171-179.
17. S.J. Liao, Beyond perturbation introduction to the Homotopy analysis method, First Edition, Chapman and Hall, CRC press, Boca Raton, (2003):336-339.
18. S.J. Liao, The proposed Homotopy analysis technique for the solution of non linear problems, Ph.D. Thesis, Shanghai Jiao Tong University,(1992):371-380.
19. V. Ananthaswamy and S.Uma Maheswari, Analytical expression for the hydrodynamic fluid flow through a porous medium, International Journal of Automation and Control Engineering, 4(2)(2015), 67-76.
20. V. Ananthaswamy and L.Sahaya Amalraj, Thermal stability analysis of reactive hydromagnetic third-grade fluid using Homotopy analysis method, International Journal of Modern Mathematical Sciences, 14(1)(2016), 25-41.
21. V.Ananthaswamy and J.Sowmyadevi, Mathematical expressions of heat transfer to MHD oscillatory flow and Homotopy analysis method, International Journal of Mathematics and its applications, Volume 4, Issue 1-D(2016), 175-187.

22. V. Ananthaswamy and T.Iswarya, Analytical expressions of the effect of radiation on free convective flow of heat and mass transfer, Nonlinear studies, 23(1)(2016), 133-147.
23. S. K. Ghoshand I. Pop, A new approach on MHD natural convection boundary layer flow past a flat plate of finite dimensions, Int. J. Heat mass transfer, 42(2005) 587-595.
24. B. Vasu, S. Rama, R Gorla, P V S N Murthy and V. R. Prasad, Free convection flow of non- Newtonian power-law plate, 20th Australasian fluid mechanics conference 5-8 (2016).
25. S. Das, B.C. Sarkar and R.N. Jana, Entropy generation in MHD free convective boundary layer flow past an inclined flat plate embedded in a porous medium with hall currents, International Journal of Computer Application (0975-8887),Vol. 84 (2013):68-75.
26. A.R. Rao and K.S. Deshikachar, Asymptotic solution for steady streaming MHD flow in a channel of a variable cross section, Int. J. Engng. Sci., 24(10)(1986), 1628.
27. Y. J. Kim, Unsteady MHD convective Heat transfer past a semi-infinite vertical plate, Int. J. Eng. Sci., 38(2000), 833-845.
28. B. Raftari, A. Yildirim, The Application of Homotopy perturbation method for MHD flows of UCM fluids above porous Stretching sheets, computer and Mathematics with Applications, 59,p.3328(2010).
29. M.M. Rashidi, The modified Differential Transform method for solving MHD Boundary- layer Equations, computer physics communications, 180,2210(2009):905-909.
30. K. Saravanakumar, V. Ananthaswamy, M. Subha, and L. Rajendran, Analytical solution of nonlinear boundary value problem for in efficiency of convective straight fins with temperature –dependent thermal conductivity , ISRN Thermodynamics, Article ID 282481(2013):1-18.
31. M. Subha, V. Ananthaswamy and L. Rajendran, Analytical solution of non-linear boundary value problem for the electrohydrodynamic flow equation, International journal of Automation and control Engineering,3(2),(2014):48-56.

Appendix: A

Basic Concept of the Homotopy analysis method

Consider the following differential equation:

$$N[u(t)] = 0 \qquad \text{(A.1)}$$

Where N is a nonlinear operator, t denotes an independent variable, $u(t)$ is an unknown function. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. By means of generalizing the conventional Homotopy method, Liao constructed the so-called zero-order deformation equation as:

$$(1-p)L[\varphi(t;p) - u_o(t)] = phH(t)N[\varphi(t;p)] \qquad \text{(A.2)}$$

where $p \in [0,1]$ is the embedding parameter, $h \neq 0$ is a nonzero auxiliary parameter, $H(t) \neq 0$ is an auxiliary function, L an auxiliary linear operator, $u_o(t)$ is an initial guess of $u(t), \phi(t:p)$ is an unknown function. It is important to note that one has great freedom to choose auxiliary unknowns in HAM. Obviously, when $p = 0$ and $p = 1,$ it holds:

$$\varphi(t;0) = u_o(t)\ text and\ \varphi(t;1) = u(t) \qquad \text{(A.3)}$$

respectively. Thus, as p increases from 0 to 1, the solution $\varphi(t;p)$ varies from the initial guess $u_o(t)$ to the solution $u(t)$.

Expanding $\varphi(t;p)$ in Taylor series with respect to p, we have:

$$\varphi(t;p) = u_o(t) + \sum_{m=1}^{+\infty} u_m(t) p^m \qquad \text{(A.4)}$$

$$u_m(t) = \frac{1}{m!} \frac{\partial^m \varphi(t;p)}{\partial p^m}\bigg|_{p=0} \qquad \text{(A.5)}$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter h, and the auxiliary function are so properly chosen, the series eqn.(A.4) converges at $p = 1$ then we have:

$$u(t) = u_o(t) + \sum_{m=1}^{+\infty} u_m(t) \qquad \text{(A.6)}$$

Differentiating the eqn.(A.2) for m times with respect to the embedding parameter p, and then setting $p = 0$ and finally dividing them by m!, we will have the so-called mth-order deformation equation as:

$$L[u_m - \chi_m u_{m-1}] = hH(t)\Re_m(\vec{u}_{m-1}) \quad \text{(A.7)}$$

Where

$$\Re_m(\vec{u}_{m-1}) = \frac{1}{(m-1)!}\frac{\partial^{m-1}N[\varphi(t;p)]}{\partial p^{m-1}} \quad \text{(A.8)}$$

And

$$\chi_m = \begin{cases} 0, m \le 1, \\ 1, m > 1. \end{cases} \quad \text{(A.9)}$$

Applying L^{-1} on both side of equation (A7), we get

$$u_m(t) = \chi_m u_{m-1}(t) + hL^{-1}[H(t)\Re_m(\vec{u}_{m-1})] \quad \text{(A.10)}$$

In this way, it is easily to obtain u_m for $m \ge 1$, at M^{th} order, we have

$$u(t) = \sum_{m=0}^{M} u_m(t) \quad \text{(A.11)}$$

When $M \to +\infty$, we get an accurate approximation of the original eqn. (A.1). For the convergence of the above method we refer the reader to Liao [15]. If the eqn.(A.1) admits unique solution, then this method will produce the unique solution.

Appendix: B

Solution of the non-linear differential eqns.(8)-(11) using the Homotopy analysis method

The analytical expressions involves in the equation (8) and (11) for $f(\eta)$ are derived using HAM method. The eqns.(6) - (7) are as follows,

$$f''' + ff'' - \beta f'^2 - Mf' = 0 \quad \text{(B.1)}$$

$$f(0) = K, f'(0) = 1, f'(\infty) = 0 \quad \text{(B.2)}$$

Where

$$\beta = \frac{2n}{n+1}, M = \frac{2\sigma B_0{}^2}{\rho c(1+n)}, \; K = \frac{V_0}{\sqrt{\frac{cv(n+1)}{2} x^{\frac{n=1}{2}}}} \quad \text{(B.3)}$$

We construct the Homotopy for the equations (B.1) as follows,

$$(1-p)(f''' - Mf') = hp(f''' + ff'' - \beta f'^2 - Mf') \quad \text{(B.4)}$$

The approximate solution of the eqn.(B.1) is as follows:

$$f = f_0 + pf_1 + p^2 f_2 + ... \quad \text{(B.5)}$$

By substituting the eqn.(B.5) into an eqn. (B.4) we get

$$(1-p)\left[(f_0''' + pf_1''' + p^2 f_2''' + ...) - M(f_0 + pf_1 + p^2 f_2 + ...)\right]$$
$$= hp\begin{bmatrix} (f_0''' + pf_1''' + p^2 f_2''' + ...) + (f_0 + pf_1 + p^2 f_2 + ...)(f_0'' + pf_1'' + p^2 f_2'' + ...) \\ -\beta(f_0' + pf_1' + p^2 f_2' + ...)^2 - M(f_0 + pf_1 + p^2 f_2 + ...) \end{bmatrix} \quad \text{(B.6)}$$

Equating the coefficients of p^0 for the equations (B.5) we get

$$p^0 : f_0''' - Mf_0' = 0 \quad \text{(B.7)}$$

$$p^1 : f_1''' - Mf_1' - hf_0 f_0'' + hf_0'^2 = 0 \quad \text{(B.8)}$$

By taking the initial solution as

$$f_0 = K\left(2 - e^{\frac{-x}{K}}\right) \quad \text{(B.9)}$$

Solving (B.6) and (B.7) using the boundary conditions (B.2), we get

$$f_1 = C_1 \frac{x^2}{2} + C_2 x + C_3 - K^3 \left[(2+M) e^{\frac{-x}{k}} + \frac{(\beta-1)}{8} e^{\frac{-2x}{K}} \right] \quad \text{(B.10)}$$

Where

$$C_1 = 0 \quad \text{(B.11)}$$

$$C_2 = -K^2 \left(2 + M + \frac{(\beta-1)}{4} \right) \quad \text{(B.12)}$$

$$C_3 = K^3 \left[2 + M + \frac{(\beta-1)}{8} \right] \quad \text{(B.13)}$$

According to Homotopy analysis method we have

$$f = \lim_{p \to 1} f(x) = f_0 + f_1 \quad \text{(B.14)}$$

Using the equations (B.8) and (B.9) in equation (B.13) we get the solution obtained in the text eqn.(8).

Appendix C:

Nomenclature

Symbol	Meaning
ν	Kinematic viscosity
ρ	Density
σ	Electrical conductivity
V_0	Porosity of the plate
C	Stretching parameter
β	Non- dimensional parameter
M	Magnetic parameter
K	Wall mass transfer parameter
u	Velocity component in *x*-direction
v	Velocity component in *y*-direction
f'	Dimensionless velocity
η	Dimensionless distance