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MATHEMATICS

OBSERVATION ON THE TERNARY QUADRATIC DIOPHANTINE EQUATION WITH THREE UNKNOWNS

$$13x^2+3y^2=640z^2$$

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ABSTRACT

The ternary quadratic equation given by is considered and searched for its many different integer solution. Seven different choices of integer solution of the above equations are presented. A few interesting relations between the solutions and special polynomial numbers are presented.

KEYWORDS: Ternary Quadratic, Integer Solutions

Notation:

$$t_{m,n}=n\left[1+\frac{(n-1)(m-2)}{2}\right]$$

$$PR_n=n(n+1)$$

$$G_n=2n-1$$

INTRODUCTION

The Diophantine equations offer an unlimited field for research due to their variety [1-3]. In particular , one may refer [4-12] for quadratic equations with three unknowns. This communication concerns with yet another interesting equation representing homogeneous equation with three unknowns for determining its infinitely many non-zero integral points. Also,few interesting relations among the solutions are presented.

METHOD OF ANALYSIS

The Quadratic Diophantine equation with three unknowns to be solved is given by,

$$13x^2+3y^2=640z^2 \tag{1}$$

Consider the linear transformation

$$\left.\begin{aligned} x &= X-3T \\ y &= X+13T \end{aligned}\right\} \tag{2}$$

Substituting (2) in (1) we get,

$$X^2+39T^2=40Z^2 \tag{3}$$

Assume $z=a^2+39b^2$ (4)

Write 40 as,

$$40=\left(1+i\sqrt{39}\right)\left(1-i\sqrt{39}\right) \tag{5}$$

Substituting (4) ,(5) in (3) and employing the method of factorization, we get

$$\left(X+i\sqrt{39}T\right)\left(X-i\sqrt{39}T\right)=\left(1+i\sqrt{39}\right)\left(1-i\sqrt{39}\right)\left(a+i\sqrt{39}b\right)^2\left(a-i\sqrt{39}b\right)^2$$

Equating the positive factor,

$$\left(X+i\sqrt{39}T\right)=\left(1+i\sqrt{39}\right)\left(a+ib\sqrt{39}b\right)^2$$

$$\left(X+i\sqrt{39}T\right)=\left(a^2-78ab-39b^2\right)+i\sqrt{39}\left(a^2+2ab-39b^2\right)$$

Equating real and imaginary parts,

$$X=a^2-78ab-39b^2$$

$$T=a^2+2ab-39b^2$$

Substituting in (2)

$$x(a,b)=-2a^2+78b^2-84ab$$

$$y(a,b)=14a^2-546b^2-52ab$$

The non-zero distinct integer solution of (1) is obtained as

$$x(a,b)=-2a^2+78b^2-84ab$$

$$y(a,b)=14a^2-546b^2-52ab$$

$$z(a,b)=a^2+39b^2$$

PROPERTIES:

- $.x(a,1)+y(a,1)-148t_{4,a}+136P_{ra}\equiv 0 \pmod 2$
- $.x(a,1)+z(a,1)-83t_{4,a}-84P_{ra}\equiv 0 \pmod 3$
- $z(a,1)+y(a,1)-67t_{4,a}-52P_{ra}-50=0$
- $y(a,1)-14z(a,1)\equiv 0 \pmod{52}$
- $7x(a,1)+y(a,1)+G_{320a}+1=0$

PATTERN:2

'40' can also be written as

$$40=\frac{(11+i\sqrt{39})(11-i\sqrt{39})}{2^2} \qquad (6)$$

Substituting (4) ,(5) &(6) in (3) and employing the method of factorization, we get

$$(X+i\sqrt{39}T)(X-i\sqrt{39}T)=\frac{(11+i\sqrt{39})(11-i\sqrt{39})(a+i\sqrt{39}b)^2(a-i\sqrt{39}b)^2}{2^2}$$

Equating the positive factor

$$(X+i\sqrt{39}T)=\frac{(11+i\sqrt{39})(a+i\sqrt{39}b)^2}{2}$$

$$(X+i\sqrt{39}T)=(11a^2-78ab-429b^2)+i\sqrt{39}(a^2+22ab-39b^2)$$

Equating real and imaginary parts of the above equation, we get

$$X=\frac{1}{2}(11a^2-78ab-429b^2)$$

$$T=\frac{1}{2}(a^2+22ab-39b^2)$$

Substituting in (2)

$$x=4a^2-72ab-156b^2$$

$$y=12a^2+104ab-468b^2$$

The non- zero distinct integral solution of (1) is obtained as

$$x(a,b)=4a^2-72ab-156b^2$$

$$y(a,b)=12a^2+104ab-468b^2$$

$$z(a,b)=a^2+39b^2$$

PROPERTIES:

- $x(a,1)+y(a,1)+16t_{4,a}-32P_{ra}\equiv 0(\mathrm{mod}\,2)$
- $x(a,1)+z(a,1)-85t_{4,a}+72P_{ra}\equiv 0(\mathrm{mod}\,3)$
- $y(a,1)+z(a,1)+91t_{4,a}-104P_{ra}+429=0$
- $x(a,1)-4z(a,1)\equiv 48(\mathrm{mod}\,72)$
- $3x(a,1)-y(a,1)+G_{160a}+1=0$

PATTERN: 3

Write the equation (3) as

$$X^2+39T^2=40Z^2\times 1 \qquad (7)$$

'1' can be written as

$$1=\frac{(7+3i\sqrt{39})(7-3i\sqrt{39})}{20^2} \qquad (8)$$

Substituting (6), (7) and (8) in (3) and employing the method of factorization, we get

$$(X+i\sqrt{39}T)(X-i\sqrt{39}T)=\frac{(7+3i\sqrt{39})(7-3i\sqrt{39})}{20^2}\frac{(11+i\sqrt{39})(11-i\sqrt{39})}{2^2}(a+i\sqrt{39}b)^2(a-i\sqrt{39}b)^2$$

Equating the positive factor

$$(X+i\sqrt{39}T)=\frac{(7+3i\sqrt{39})}{20}\frac{(11+i\sqrt{39})}{2}(a+i\sqrt{39}b)^2$$

$$(X+i\sqrt{39}T)=\frac{(-40a^2+1560b^2+3120ab)+i\sqrt{39}(40a^2-1560b^2-80ab)}{40}$$

Equating real and imaginary part of the above equation, we get

$$X=-a^2+39b^2+78ab$$

$$T=-a^2-39b^2-2ab$$

Substituting in (2)

$$x=-4a^2+156b^2-72ab$$

$$y=12a^2-468b^2-104ab$$

The non-zero distinct integral solution of (1) is obtained as

$$x(a,b)=-4a^2+156b^2-72ab$$

$$y(a,b)=12a^2-468b^2-104ab$$

$$z(a,b)=a^2+39b^2$$

PROPERTIES:

- $x(a,1)+y(a,1)-184t_{4,a}+352P_{ra}+312=0$
- $.x(a,1)+z(a,1)-69t_{4,a}+72P_{ra}\equiv 0(\mathrm{mod}\,5)$
- $.y(a,1)+z(a,1)-117t_{4,a}+104P_{ra}\equiv 0(\mathrm{mod}\,3)$
- $3x(a,1)+y(a,1)+G_{160a}+1=0$
- $y(a,1)-12z(a,1)\equiv 0(\mathrm{mod}\,104)$

PATTERN:4

(3) can also be written as

$$(X+Z)(X-Z)=39(Z+T)(Z-T) \qquad (9)$$

Case: 1

(9) can be written in the form of ratio as

$$\frac{(X+Z)}{13(Z+T)}=\frac{3(Z-T)}{(X-Z)}=\frac{\alpha}{\beta},\beta\neq 0 \qquad (10)$$

which is equivalent to the system of double equation as

$$\left.\begin{aligned}X\beta-13\alpha T+Z(\beta-13\alpha)=0\\-X\alpha-3\beta T+Z(\alpha+3\beta)=0\end{aligned}\right\} \qquad (11)$$

Solving (11) by the method of cross multiplication, we get

$$\left.\begin{aligned}X&=-13\alpha^2-78\alpha\beta+3\beta^2\\T&=-13\alpha^2+2\alpha\beta+3\beta^2\\z&=-13\alpha^2-3\beta^2\end{aligned}\right\} \qquad (12)$$

Substituting (12) in (2) the non-zero distinct integer solution of (1) are given by

$$x(\alpha,\beta)=26\alpha^2-84\alpha\beta-6\beta^2$$

$$y(\alpha,\beta)=-182\alpha^2-52\alpha\beta+42\beta^2$$

$$z(\alpha,\beta)=-13\alpha^2-3\beta^2$$

PROPERTIES:

- $x(\alpha,1)+y(\alpha,1)+20t_{4,a}+136P_{ra}\equiv 0(\mathrm{mod}\,2)$
- $x(\alpha,1)+y(\alpha,1)-97_{4,a}+84P_{ra}\equiv 0(\mathrm{mod}\,3)$
- $y(\alpha,1)+z(\alpha,1)+143_{4,a}+52P_{ra}-1=0$
- $x(\alpha,1)+2z(\alpha,1)\equiv 7(\mathrm{mod}\,84)$
- $7x(\alpha,1)+y(\alpha,1)+G_{320\alpha}+1=0$

Case: 2

(9) can also be written in the form of the ratio as

$$\frac{X+Z}{3(Z+T)}=\frac{13(Z-T)}{(X-Z)}=\frac{\alpha}{\beta},\beta\neq 0$$

which is equivalent to the system of double equation as

$$\left.\begin{array}{l} X\beta-3\alpha T+Z(-3\alpha+\beta)=0 \\ -X\alpha-13\beta T+Z(13\beta+\alpha)=0 \end{array}\right\}$$

Solving (13) by the method of cross multiplication, we get

$$\left.\begin{array}{l} X=-3\alpha^2-78\alpha\beta+13\beta^2 \\ T=-3\alpha^2+2\alpha\beta+13\beta^2 \\ z=-3\alpha^2-13\beta^2 \end{array}\right\}$$

substituting (14) in (2), the non-zero distinct integral solution of (1) are given by

$$x(\alpha,\beta)=6\alpha^2-84\alpha\beta-26\beta^2$$

$$y(\alpha,\beta)=-42\alpha^2-52\alpha\beta+182\beta^2$$

$$z(\alpha,\beta)=-3\alpha^2-13\beta^2$$

PROPERTIES:

- $x(\alpha,1)+y(\alpha,1)-100t_{4,a}+136P_{r,a}\equiv 0(\mathrm{mod}\,2)$
- $.x(\alpha,1)+z(\alpha,1)-87t_{4,a}-84P_{r,a}+23=0$
- $.y(\alpha,1)+z(\alpha,1)-7t_{4,a}+52_{r,a}\equiv 0(\mathrm{mod}\,5)$
- $x(\alpha,1)+2z(\alpha,1)\equiv 52(\mathrm{mod}\,84)$
- $7x(\alpha,1)+y(\alpha,1)+G_{320\alpha}+1=0$

Case: 3

(9) can be written in the form of the ratio as

$$\frac{X+Z}{39(Z+T)}=\frac{(Z-T)}{X-Z}=\frac{\alpha}{\beta},\beta\neq 0$$

which is equivalent to the system of double equation as

$$\left.\begin{array}{l} X\beta-39\alpha T+Z(-39\alpha+\beta)=0 \\ -\alpha X-\beta T+Z(\alpha+\beta)=0 \end{array}\right\}$$

(15)

solving (15) by the method of cross multiplication, we get

$$\left.\begin{array}{l} X=-39\alpha^2-78\alpha\beta+\beta^2 \\ T=-39\alpha^2+2\alpha\beta+\beta^2 \\ z=-39\alpha^2-\beta^2 \end{array}\right\} \qquad (16)$$

substituting (16) in (2), we obtained the non-zero distinct integral solution of (1) are given by

$$x(\alpha,\beta)=78\alpha^2-84\alpha\beta-2\beta^2$$

$$y(\alpha,\beta)=-546\alpha^2-52\alpha\beta+14\beta^2$$

$$z(\alpha,\beta)=-39\alpha^2-\beta^2$$

PROPERTIES:

- $x(\alpha,1)+y(\alpha,1)+332t_{4,a}+136P_{r,a}\equiv 0(\mathrm{mod}\,3)$
- $x(\alpha,1)+z(\alpha,1)-123t_{4,a}+84P_{r,a}\equiv 0(\mathrm{mod}\,3)$
- $.y(\alpha,1)+z(\alpha,1)+533t_{4,a}+52P_{r,a}-13=0$ (13)
- $y(\alpha,1)-14z(\alpha,1)\equiv 24(\mathrm{mod}\,52)$
- $7x(\alpha,1)+y(\alpha,1)+G_{320\alpha}+1=0$

Case:4

(9) can be written in the form of the ratio as

$$\frac{X+Z}{Z+T}=\frac{39(Z-T)}{X-Z}=\frac{\alpha}{\beta},\beta\neq 0$$

which is equivalent to the system of double equation as (14)

$$\left.\begin{array}{l} X\beta-\alpha T+Z(-\alpha+\beta)=0 \\ -\alpha X-39\beta T+Z(\alpha+39\beta)=0 \end{array}\right\} \qquad (18)$$

Solving (18) by the method of cross multiplication, we get

$$\left.\begin{array}{l} X=-\alpha^2-78\alpha\beta+39\beta^2 \\ T=-\alpha^2+2\alpha\beta+39\beta^2 \\ z=-\alpha^2-39\beta^2 \end{array}\right\} \qquad (19)$$

Substituting (19) in (1), the non-zero distinct integral solution of (1) are given by

$$x(\alpha,\beta)=2\alpha^2-84\alpha\beta-78\beta^2$$

$$y(\alpha,\beta)=-14\alpha^2-52\alpha\beta+546\beta^2$$

$$z(\alpha,\beta)=-\alpha^2-39\beta^2$$

PROPERTIES:

- $.x(\alpha,1)+y(\alpha,1)-124t_{4,a}-136P_{ra}\equiv 0(\bmod 2)$
- $x(\alpha,1)+z(\alpha,1)-85t_{4,a}+84P_{ra}+117=0$
- $y(\alpha,1)+z(\alpha,1)-37t_{4,a}+52P_{ra}\equiv 0(\bmod 3)$
- $y(\alpha,1)-14z(\alpha,1)\equiv 0(\bmod 52)$
- $7x(\alpha,1)-14z(\alpha,1)+G_{320\alpha}+1=0$

CONCLUSION:

In this paper, an attempt has been made to obtain non-zero distinct integer solutions to the ternary quadratic diophantine equation representing $13x^2+3y^2=640z^2$ homogenous cone. As the Diophantine equations are rich in variety, one may search for integer solutions to higher degree Diophantine equations with multiple variables along with suitable properties.

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